Game Theory: Employing the Prisoner's Dilemma to Enhance Interdisciplinary Learning

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ABSTRACT

This article describes game theory from a restricted perspective and elucidates how it facilitates interdisciplinary learning. Initially, game theory's basic components and the Prisoner's Dilemma, used throughout the article to analyze game theory, are described. Next, the normal (or strategic) form and the extensive form for representing game theory are illustrated. Then a brief discussion of, and the role of, a Nash Equilibrium is provided. This is followed by a scenario involving current events to synthesize the aforementioned discussions of game theory. Finally, we discuss both the link between interdisciplinary learning and game theory and its implications.

Analysis of behavior in strategic situations--whereby one individual's choices reflect success or failure, depending upon others' choices and vice versa--is clearly not a recent or novel idea. However, the ability to substantively describe such behavior mathematically is of rather recent vintage—beginning, realistically, in the 1940s with game theory (McCain, 2004). Although, the theory itself has been modified and extended, thereby improving its functionality, it is still not free of problems and shortcomings. On the other hand, one feature that remains constant throughout game theory's history is the need to frequently integrate knowledge from various disciplines in order to make prudential choices and to maximize a successful learning experience. In this article, then, we discuss game theory from a restricted perspective (see Stanford Encyclopedia of Philosophy, 2007, for an in-depth analysis of game theory) and show how it facilitates learning. First, we briefly define the basic components of game theory. We then proceed to describe the Prisoner's Dilemma, arguably the most popular game used to demonstrate game theory, which will be used throughout the article to analyze game theory. This is followed by an illustration of the two techniques used in the literature to represent games: normal (or strategic) form and extensive form. Next, a brief discussion of, and the role of, a Nash Equilibrium, as it relates to the Prisoner's Dilemma, is provided. A scenario involving current events is then used to synthesize the aforementioned discussions of game theory. Finally, we discuss the link between interdisciplinary learning and game theory, followed by implications.

A Brief Definition of Game Theory

John von Neumann and Oskar Morgenstern's (1944) publication of *Theory of Games and Economic Behavior* ushered in modern game theory as we know it today. According to the Stanford Encyclopedia of Philosophy (2007), game theory is a very powerful tool for mathematically analyzing games by studying how rational players' strategic interactions generate outcomes relative to the players' preferences, or utilities, none of which the players may have intended. A game in this context implies a contest in which two players compete by utilizing rules that determine how the game is played and who wins. A payoff—a reward earned by the winner—is an integral part of the game, too. Games also entail strategies, which are rules for determining a player's move(s) for winning the game or maximizing his/her payoff. Though initially applied to economics, it has since then been employed in many other disciplines (see Stanford Encyclopedia of Philosophy, 2007, for an in-depth analysis of this topic). Having briefly defined game theory, we now move on to the two techniques used in the literature to represent games: normal (or strategic) form and extensive form.

Representation of Games

The Prisoner's Dilemma (thereafter PD), arguably the most famous game, is an eminent example that will be used to illustrate both forms of games (Rasmusen, 2006). After describing the nature of the Prisoner's Dilemma, it will then be represented by using the first technique—the normal (or strategic) form, followed by the other technique—the extensive form.

Prisoner's Dilemma

A description of the Prisoner's Dilemma (McCain, 2004; Rasmusen, 2006; Stanford Encyclopedia of Philosophy, 2007) is encapsulated in the following scenario: Two suspects, presumed by police to have collaborated in a jewelry heist, are arrested, despite the police's inability to obtain a jury's conviction due to insufficient admissible evidence. However, ample evidence exists for convicting both prisoners on a lesser charge, which carries a two-year jail sentence. Based on the aforementioned, each prisoner, who is interrogated separately, but simultaneously, is offered the following by the Chief Inspector: "Should you confess and implicate your accomplice, and your accomplice also does not confess, then you'll be set free while your accomplice serves 15 years. On the other hand, if your accomplice confesses, but you don't, you'll serve 15 years, and your accomplice will be set free. Should both of you confess, however, each of you will serve 7 years. And, should neither of you confess, each of you will serve 2 years for the lesser offense." Figure 1 below designates Bob's and Susan's "payoffs" as jail time in terms of years (Gaus [n.d.]; Stanford Encyclopedia of Philosophy, 2007).

		Susan: Confess	Susan: Refuse
Bob	Confess	7,7	0,15
	Refuse	15,0	2,2

Figure1. PD in prison time.

Normal (Strategic) Form

According to Figure 1 above, each cell in the matrix represents each player's payoff for each combinatorial action. It should be noted that Bob can move only top to bottom or bottom to top via columns, while Susan can move only left to right or right to left via rows. Additionally, the first and second numbers of each pair represent Bob's and Susan's payoffs, respectively. For example, assuming you take on Bob's role, a confession from both you and Susan would result in a payoff of 7 years for each of you, which is reflected in the upper-left cell (7,7). However, a payoff of 2 years would be meted out for each of you should neither you nor Susan confess, which is reflected in the lower-right cell (2,2). On the other hand, should you confess, but not Susan, you would be assigned a payoff of 0 years (set free), and Susan would be assigned 15 years as reflected in the upper-right cell (0,15). Finally, according to the lower-left cell (15,0), should you refuse to confess, but not Susan, you would receive 15 years, while Susan would receive 0 years [set free] (Stanford Encyclopedia, 2007).

Upon reflection, after the Chief Inspector's offer, both players will compare their respective columnar payoffs relative to each of their partner's possible actions. For example, Bob may realize that should Susan decide to confess and he decides to confess, that both of them will serve 7 years (7,7). Also, should Susan confess, but he decides to refuse, that he will receive 15 years, while Susan serves 0 years, or set free, (15,0). Hence, Bob realizes that if Susan confesses, that he is better off confessing, too because 7 (7,7) is less than 15 (15,0). In contrast, should Susan refuse, but Bob confesses, than Bob will serve 0 years, or be set free, while Susan serves 15 years (0,15). And should Susan refuse, and Bob refuses, too, than both will serve 2 years (2,2). Once again, Bob realizes that should Susan opt to refuse that he is again better off confessing because this time 0 (0,15) is better than 2 (2,2). Bob now fully understands that he is better off confessing regardless of what Susan does; therefore, confessing in this context is Bob's dominant strategy (7,7), while refusing is his dominated strategy because he should not refuse to confess. Thus, Bob may just as well delete the entire bottom row (left-lower corner [15.0] and right-lower corner [2.2]) because it is his dominated strategy, which works against him. Susan's thinking will parallel Bob's thinking. She will realize that should Bob confess and she does, too, that each would serve 7 years (7,7), and that should Bob confess, but she refuses, that she will serve 15 years and Bob will serve 0 years, or be set free (0, 15). Thus, Susan sees that she is better off confessing regardless of what Bob decides to do because 7 (7,7) is less than 15 (0,15). Now should Bob refuse and she confesses, that she will receive 0 years, or be set free, and Bob will serve 15 years (15,0). And should Bob refuse and she refuses, that each would serve 2 years (2,2). Once again, Susan is better off confessing because 0 (15,0) is less than 2 (2,2). Thus, just as for Bob, confessing turns out to be Susan's dominant strategy (7,7), too, because confessing nets her better results regardless of what Bob does. Moreover, refusing is also Susan's dominated strategy because she should not refuse to confess. Hence, Susan needs to delete the entire right column (upper-right corner [0,15] and lower-right column [2,2]) because it is her dominated strategy, which works against her. Consequently, when both the entire lower row and the entire right column are deleted or omitted, we are left with only the upper-left corner (7,7), which is the solution or equilibrium to this PD example—or its dominant strategy. Actually, any temptation to deviate from this strictly dominant strategy is totally eliminated because both participants are mutually aware of this. Accordingly, both participants will serve a 7-year prison term because each will confess to the crime. In essence, a joint confession is perceived as the game's solution—its basin of attraction or convergence point—precisely because each participant is presumed to be economically rational, i.e., higher payoffs are preferable to lower payoffs (Gaus [n.d.]). We will return to these three pivotal game theory concepts—dominant strategy, solution, and equilibrium—when we discuss Nash equilibria.

At this point, we have merely narrated Bob and Susan's reactions to each other's possible moves within the context of jail time. Staying out of jail and/or minimizing jail time is what we assumed Bob and Susan are focusing on. If, however, we truly wish to demonstrate that confessing is the most rational thing for them to do, we must address their utility functions, i.e. their preferences over outcomes. For this purpose, the following ordinal utility functions can be generated that would map the rankings of outcomes, or ordered preferences, for both Bob and Susan onto numbers: 4 = 0 years, 3 = 2 years, 2 = 7 years, and 1 = 15 years. We notice that the higher the number, the more preferred the outcome. More specifically, we say that these numbers represent an ordinal utility function because they measure only order—not magnitude, which would render them cardinal utility functions. As can also be seen, there is an inverse relationship between the utility function numbers and prison time, i.e., the higher the ranking, the less jail time is associated with it. Figure 2 below represents Bob's and Susan's utility functions, which reflect a preference for less jail time over more jail time (Gaus [n.d.]).

		Susan: Confess	Susan: Refuse
Bob	Confess	2,2	4,1
Bob	Refuse	1,4	3,3

Figure 2. The general ordinal utility form of a PD.

Once again, even though both Bob and Susan would have preferred utility 3 (refuse/refuse), they still end up instead with utility 2 (confess/confess). Thus, in PD examples, the truly best outcome is not necessarily the end result—behavior wise. The impact of this intrinsic feature of the PD on game theory will be elaborated when equilibrium concepts associated with game theory are discussed. In the next section, this specific PD game will be extended to contrast its current strategic-form with its extensive-form.

Extensive Form

When the PD is treated as a game in its strategic-form, collusive agreement is implicitly obviated by the prisoners' simultaneous actions because both prisoners wish to avoid their worst payoff or outcome. However, we may conjecture that if the participants' moves are not simultaneous, that that could or would change the outcome. Unfortunately, the extensive-form of this PD game will render this intuition's conclusions false and misleading. This will be demonstrated by illustrating the PD game in its extensive form through game-trees and analytical methods associated with them.

In any game-tree, the following concepts/components are critical for analyzing extensive-form games: a node is defined as any point whereby an action is taken by a player; an initial node is the point that marks the game's first action; a terminal node, by contrast, represents an outcome and terminates a game, if such a node is reached; a subgame is an assemblage of nodes and branches uniquely descending from a single node; a payoff is an ordinal utility number or function that a player is assigned, relative to an outcome; an outcome is characterized by a set of payoffs assigned to each player in a game, and a strategy instructs players regarding which action is appropriate at any of the tree's nodes where possible choices could be executed (Dixit, A.K. & Nalebuff, B.J., 1993; Gaus [n.d.]; Standford Encyclopedia, 2007).

Initially Bob and Susan, according to Figure 2, had both opted simultaneously to confess as shown in the top-left corner of the matrix (2,2); however, upon consulting the matrix, both of them realized that the bottom right-corner of the matrix (3,3) offers a better outcome; thus, both of them agree to cooperate, with Bob committed to refuse first, followed then by Susan's reciprocation. This latter strategy to honor the agreement is referred to as "refuse" and is denoted by "R" in the tree below (see Figure 3); in contrast, the strategy to refuse to honor the agreement is referred to as "confess" and will be denoted by "C" in the tree below (see Figure 3).

Looking at Figure 3, you take the role of Bob (Participant I), and your partner takes on the role of Susan (Participant II)—just as was done in Figure 1. "C" stands for "confession," and "R" stands for "refuse." The numbers 1, 2, and 3 denote each node in the tree. Each of the nodes along the bottom of the tree is a terminal node ([2,2], [4,1], [1,4], and [3,3]). Moreover, each terminal node represents a potential outcome corresponding with assigned payoffs—as illustrated

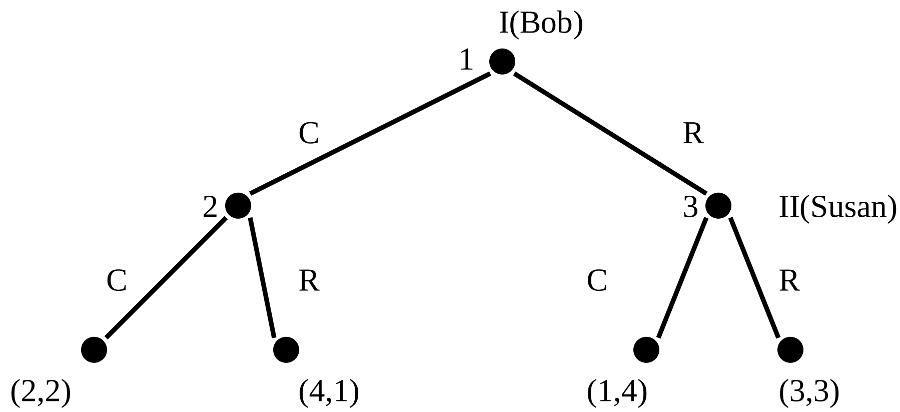

Figure 3. PD in extensive form.

in the strategic-form game, i.e., in each set, the payoffs for Bob (I) always appear first while Susan's (II) payoffs always appear second. Nodes 1, 2, and 3, respectively, have structures gravitating from them, which represent sub-games (Standford Encyclopedia, 2007).

In strategic-form representation games, participants indulge in simultaneous moves; however, in extensive-form games, they do so sequentially. With this in mind, initially, backward-induction analysis is used with each of the sub-games arising last in our sequence of play. Accordingly, should Susan (II) select to play the sub-game gravitating from node 3, she will need to choose either a payoff of 4 (1,4) or a payoff of 3 (3,3). Susan's (II) payoff is represented by the second number in both terminal nodes [(1,4) and (3,3)] gravitating from node 3. Susan (II) realizes immediately that playing C (1,4), confess, generates a higher payoff than R (3,3), refuse, because 4 (1,4) is greater than 3 (3,3). Hence, the entire subgame may be replaced with the payoff (1,4) directly assigned to node 3, because this outcome will materialize should the game reach this node. On the other hand, Susan (II) confronts a different task should she opt for the subgame gravitating from node 2, leaving her to choose between a payoff of 2 (2,2) or a payoff of 1 (4,1). Playing C clearly provides Susan (II) with a higher payoff of 2, confess, than would a payoff of 1, refuse. Thus, the payoff of 2(2,2), confess-confess, may consequently be directly assigned to node 2. Our next move focuses on the sub-game gravitating from node 1, which incidentally is isomorphic with respect to the whole game because every game is inherently a subgame of itself—much as all sets—according to set theory—are subsets of themselves (Erdelsky, 2007). Bob (I), at this point, must choose either outcome (2,2), which has a payoff of 2, confess, or outcome (1,4), which has a payoff of 1, refuse. Based on the first number of both sets, Bob (I) immediately realizes that playing C generates a higher payoff, because 2 is greater than 1, and opts to confess. Hence, in the end, both strategic-form and extensive-form representations yield identical outcomes—a confession (Gaus [n.d.]; Stanford Encyclopedia of Philosophy, 2007).

In analyzing why both forms of games yielded identical results, Bob (I) realized that playing R (refuse to confess) at node 1 maximized Susan's (II) utility by suckering Bob (I) through playing C, which occurs at node 3 on the tree. Bob (I) would have been left with a payoff of 1 (a 15-year prison term) [1,4], which could be obviated only by opting for C (by defecting from the original agreement) at the outset.

It should be noted that although the simultaneous and sequential versions of the PD in our aforementioned examples yielded similar outcomes that this will by no means always be the case. In the next section, Nash equilibria will be briefly discussed as it relates to the PD.

Nash Equilibria vis-à-vis the PD

According to the Prisoner's Dilemma, (see Figure 2), the outcome depicted by (2,2), indicating joint confession, was declared as the game's solution or equilibrium. More precisely, in this context, we say that this game has a unique solution (joint confession) or a unique Nash equilibrium (henceforth 'NE'). In essence, a NE demonstrates in this case that both prisoners would have been better off had it not been for rational self-interest. In other words, we say that for Bob and Susan to be in NE, that Bob is executing his best decision available to him, taking Susan's decision into account, and Susan is executing her best decision available to her, taking Bob's decision into account.

Using Figure 2, we can demonstrate that (2,2) is the game's NE by showing that Bob and Susan have no incentive to deviate from (2,2), which generates their strategy profile's best payoffs (Rasmusen, 2006). Focusing initially, then, on only the upper-left corner (2,2) of Figure 2, let us start off with Susan: Should Susan change her mind, but not Bob, Susan would go from confess (2,2) to refuse (4,1); thus she would go from 2 to 1, making it a worse outcome. But if Bob, instead, changes his mind, but not Susan, Bob would go from confess (2,2) to refuse (1,4); hence, he would go from 2 to 1, making it a worse outcome. Thus, neither has an incentive to change.

The question arises: Can we be sure that (2,2) is indeed the only NE or solution in this game? And the answer is yes; accordingly, let us look at the three remaining corners of Figure 2. Let us look specifically at the bottom-left corner (1,4). Should Bob change his mind from refuse (1,4) to confess (2,2), he would go from 1 to 2, making it a better outcome and an incentive for him to change, which violates the whole philosophy behind a NE. Should Susan change her mind from confess (1,4) to refuse (3,3), she would go from 4 to 3 and would be in worse shape.

Upon looking at the upper-right corner (4,1), should Susan change her mind from refuse (4,1) to confess (2,2), she would go from 1 to 2, which would improve her payoff and provide her with an incentive to change, which does not harmonize with NE's philosophy. Should Bob, on the other hand, change his mind from confess (4,1) to refuse (3,3), he would go from 4 to 3 and be in worse shape.

Finally, let us look at the bottom-right corner (3,3). Should Bob change his mind from refuse (3,3) to confess (4,1), he would go from 3 to 4 and improve his payoff and have an incentive to change, which does not harmonize with NE's philosophy. Also, should Susan change her mind from refuse (3,3) to confess (1,4), she, too, would have an incentive to change. Hence, both incentives violate the philosophy behind NE. Accordingly, as has been demonstrated, this PD game does indeed exhibit a unique solution (joint confession) or a unique Nash equilibrium. However, it is quite possible for games to have either more than one NE or none (Rasmusen, 2006). In the next section, a scenario is provided for purposes of applying the PD to a current issue that has global implications.

Scenario

This section focuses on a scenario that intrinsically integrates history, economics, government/political science, and ethics. More specifically, it converges on the tension that has existed between India and Pakistan since their independence from Britain in 1947. Since the inception of their independent status, control of Kashmir, especially regarding valuable water resources, has resulted in one of the most protracted and violent conflicts the world has seen. Moreover, both countries currently possess nuclear weapons and a relatively large stockpile of conventional weapons (Schofield, 2010).

Figure 4 below provides us with an ordinal ranking of the following possible payoffs for each country regarding the current state of affairs, with "4" representing the most preferred payoff and "1" representing the least preferred payoff. Hence, according to Figure 4, India's worst payoff (1,4) occurs when Pakistan builds bombs but not India, and Pakistan's worst payoff (4,1) occurs when India builds bombs but not Pakistan. In contrast, (3,3) represents both country's ideal payoff because neither would build bombs; instead, the money that would otherwise go into the production of nuclear weapons would instead go into significantly better investment for their people. However, (2,2), upon closer inspection, is a dominant strategy because regardless of what India does, Pakistan is better off building bombs, and whatever Pakistan does, India is better off building bombs, too. Although it is logical to conclude that both countries would be better off should neither choose to build bombs (3,3), a problem arises because this strategy is unstable. The payoff (2,2), however, is stable because it is in equilibrium; in fact, it is a NE because neither country's payoff can be improved by unilaterally switching to any other available strategy.

		Pakistan: Don't Build Bombs	Pakistan: Build Bombs
India	Don't Build Bombs	3,3	1,4
	Build Bombs	4,1	2,2

Figure 4. The PD game as an arms race.

It should be emphasized that both India and Pakistan spend inordinate amounts of their budgets on equipping themselves with such large quantities of sophisticated conventional weapons, that that alone could potentially deter both countries from engaging in, or prolonging, a perceived nuclear arms race. Also, this diversion of monies from human development—given how many poor people live in both countries—to military buildup may also minimize the odds of engaging in a nuclear conflagration. Nonetheless, we should also keep in mind that India needs to protect itself from China, who has previously gone to war with India. Moreover, Pakistan has received military aid from China and the U.S.A. In addition, India has received military aid from Russia and the U.S.A. Finally, America's involvement in Afghanistan is also creating additional friction between the two arch rivals—India and Pakistan. Thus, it is quite possible that ongoing friction between both countries could erupt so much so that animosity and distrust/fear of each other could catalyze a nuclear option (Schofield, 2010). In the next section, the mechanisms that enable game theory, and more specifically the PD, to enhance interdisciplinary learning are discussed.

Enhancing Interdisciplinary Learning via Game Theory

According to McNeil (1992), “schemata are the reader’s concepts, beliefs, expectations, processes--virtually everything from past experiences—that are used in making sense of things and actions” (p. 20). In reading, comprehension is made possible through the interaction of the reader’s schemata with the author’s input. Readers, according to McNeil, possess three types of schemata to help them grasp what they are reading: (1) domain schemata, which are topic/subject specific knowledge, (2) general world knowledge, which enhances a reader’s ability to infer appropriately while reading and familiarizes her/him with knowledge common to numerous domain specific situations, and (3) knowledge of rhetorical structures, which reduces the reader’s task of efficiently processing both narrative and expository text.

Schemata, in other words, enable us to grasp what we are reading because they facilitate assimilation and accommodation of new, or radically new, information through slots; slots, in turn, are an important reason why all schemata are dynamic—not static—and why all schemata are embedded within other schemata. Schemata also function in other ways: they enable readers to ascertain what is important, to infer and elaborate, to summarize by distinguishing amongst ideas at different levels of importance, and to improve memory by efficiently storing knowledge for easier future retrieval purposes (McNeil, 1992).

Game theory, consequently, is an ideal strategy or technique for learning information precisely because readers are required to utilize all of the aforementioned functions of schemata in the course of trying to solve a given problem, assuming a solution actually exists; moreover, numerous game theory scenarios simultaneously integrate various disciplines, which compel readers to resort to deep processing of information. Hence, students must actively participate, singly or in groups, when trying to understand topics selected for game theory analysis which “appear” to be more easily grasped than meets the eye due to unforeseen complexities and complications. This is precisely why game theory has so much to offer readers who wish to be adept at analyzing behavior. While game theory, using the Prisoner’s Dilemma, undoubtedly poses information processing demands, it could be used beginning at least at the 9th grade in high school, focusing initially on a single discipline, then gradually integrating multiple disciplines.

Implications

The need to analyze multitudinous situations that affect us locally or globally and that impact our past, present, or future, has occurred throughout human history. However, it is only relatively recently that we have been able to do so rigorously through powerful and resourceful mathematical analysis via game theory. Today, game theory is used to analyze behavior in virtually every discipline and continues to make inroads. This new and powerful approach is ostensibly not perfect, however, because attempting to meticulously and perfectly map out human (or any other organism’s) behavior is clearly out of the question. Nonetheless, the results derived thus far from game theory—and specifically from PD for purposes of this article—have enabled us to see much further than we have been able to in the past.

In addition, game theory is ideally and intrinsically suited for facilitating interdisciplinary learning not only because analysis of human behavior entails an array of disciplines (e.g., science, economics, mathematics, ethics, biology, and probability), but also because this rigorous analysis renders such behavior isomorphic at micro and macro levels. Thus, in the final analysis, it is this isomorphism that enables our schemata to make it possible for game theory's interdisciplinary nature to enhance learning, deep processing of information, and transfer of learning. Given the global problems confronting us today (e.g., ecological disasters, energy crisis, global financial meltdowns, global warming, influenza pandemics, and terrorism), the use of game theory at grade appropriate levels should serve the education community—and the community at large—quite well by generating potentially more independent problem solving thinkers and analyzers of behavior—a much needed commodity in today's world.

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